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A lattice Boltzmann algorithm for fluid–solid conjugate heat transfer ☆

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Abstract

A lattice Boltzmann algorithm for fluid–solid conjugate heat transfer is developed. A new generalized heat generation implement is presented and a "half lattice division" treatment for the fluid–solid interaction and energy transport is proposed, which insures the temperature and heat flux continuities at the interface. The new scheme agrees well with the classical CFD method for predictions of flow and heat transfer in a heated thick-wall microchannel with less mesh number and less computational costs.

Keywords: Lattice Boltzmann method; Conjugate heat transfer with heat generation; Fluid–solid interaction

1. Introduction

In the past two decades, the micro- and nanosystems have become of great interest [1–3] due to their important and promising applications in lab-on-a-chip [4], micro-TAS [5], and micro fuel cells [6]. The physics of fluid and heat transfer has been a very hot topic [7–10]. When a fluid flows in micro-geometries, such as in micro channels or in porous media, the heat transfer between fluid and solid boundaries can hardly be described using the experiential formulas in convectional heat transfer theories. Few experiments can result in accurate values of the heat transfer coefficient at micro- and nanoscales though many efforts have been focused on them [9,10]. Numerical analysis also meets great problems for complex geometry boundary conditions using the conventional CFD methods, such as finite difference and finite volume methods. The simple boundary implements, such as constant wall temperature or constant heat flux, are not suitable for microfluidics any more, because heat conduction in solids also plays a very important role in the overall heat transfer [8,11,12]. A fluid–solid conjugate heat transfer condition is the reasonable boundary condition at the interface, which is restricted by temperature and heat flux continuities. However such an interface condition brings much additional computational cost which is nearly unacceptably huge for classical CFD methods in complex geometries [9,11,12].

Lattice Boltzmann method, a mesoscopic statistics based method, has been used to model the flow behavior in complex geometries due to its easy implements for complex fluid–solid boundary conditions [13–18]. Since the lattice Boltzmann method is also successfully applied in fluid flow under the effect of electric field [19–23] or magnetic field [24–26], hopefully it can also deal well with the conjugate heat transfer process. In fact, various models have been built in the lattice Boltzmann method to model the thermohydrodynamics since 1993 [27–33]. In the early works a single distribution function model was introduced into the lattice Boltzmann method to simulate the Rayleigh–Bénard convection, however, with severe numerical instability and temperature variation limited to a narrow range [27–29]. To overcome these defects a double distribution function model was therefore developed [30–32], which contains a density distribution function to simulate hydrodynamics (fluid flow) and an internal energy distribution function to simulate the thermodynamics (heat transfer). He et al. [32] proved that the energy double distribution model can correctly describe viscous heat dissipation and compression work done by the

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Nomenclature

c	lattice streaming speed	$\mathrm{m\,s^{-1}}$	
c_s	speed of sound	$\mathrm{m\,s^{-1}}$	
c_p	specific heat capacity	$\mathrm{J\,kg^{-1}K^{-1}}$	
f_α	density distribution function		
f_α^{eq}	equilibrium distribution function of f_α		
g_α	energy distribution function		
g_α^{eq}	equilibrium distribution function of g_α		
P	pressure	Pa	
$\dot{Q}$	heat generation per unit volume	$\mathrm{W\,m^{-3}}$	
T	temperature	K	
$\mathbf{u}$	velocity vector	$\mathrm{m\,s^{-1}}$	

Greek symbols

τ_v	relaxation time for f_α		
τ_g	relaxation time for g_α		
μ	dynamic viscosity	$\mathrm{kg\,m^{-1}\,s^{-1}}$	
ν	kinematic viscosity	$\mathrm{m^2\,s^{-1}}$	
λ	thermal conductivity	$\mathrm{W\,m^{-1}K^{-1}}$	
δ_x	lattice spacing	m	
δ_t	time step	s	
ρ	density	$\mathrm{kg\,m^{-3}}$	

Subscripts

int	interface
s	solid
f	fluid

pressure. However, He's method is too complicated to use so that several simplified versions has been presented in the past three years. Peng et al. [33] gave a very simple internal energy function evolution with neglectable heat dissipation and compression work done by the pressure. Shi et al. [34] presented a similar thermal lattice Bhatnagar–Gross–Krook (BGK) model for fluid flow with viscous heat dissipation. Xuan et al. [35] even used a two-dimensional four speeds (D2Q4) model to study the heat transfer in nanofluids. These methods have been applied in various heat transfer problems without external heat sources [33–35]. The heat source implement is another problem in lattice Boltzmann method. Although Jiaung et al. [36,37] presented an approach to deal with the heat source in heat conduction, which is similar to add an external force term in fluid flow simulation, nevertheless, it is difficult to be used in convectional heat transfer simulations. To date, no generalized heat source implement has been proposed for the thermal lattice Boltzmann method.

In this work, we present a generalized heat source implement for the thermal lattice Boltzmann method, then the interface boundary implements are described in detail for the fluid–solid conjugate heat transfer problem. After validated by some benchmarks, the present model is applied to simulate the heat transfer process where the fluid flows in a heated thick–wall duct. Both the accuracy and computational efficiency are compared with the classical CFD method.

2. Mathematical model and numerical method

2.1. Governing equations

Consider a two-dimensional fluid–solid conjugate heat transfer problem. If the fluid is incompressible, the flow is laminar, and the physical properties of the fluid and solid are independent on temperature, the governing equations for flow and heat transfer can be written as follow [38,39].

Continuity equation

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

Momentum equation

$$\rho_f \frac{\partial \mathbf{u}}{\partial t} + \rho_f \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla P + \mu \nabla^2 \mathbf{u} \tag{2}$$

Energy equation for fluid

$$(\rho c_p)_f \left(\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right) = \lambda_f \nabla^2 T + \dot{Q}_f \tag{3}$$

Energy equation for solid

$$(\rho c_p)_s \frac{\partial T}{\partial t} = \lambda_s \nabla^2 T + \dot{Q}_s \tag{4}$$

where subscript f represents fluid and s represents solid, $\mathbf{u}$ is velocity vector and T is temperature, ρ is the density, λ is the thermal conductivity, c_p is the specific heat capacity, and $\dot{Q}$ is the heat generation per unit volume.

The interface conditions are needed for solving the governing equations in classical CFD methods, which are non-slip on the solid wall, temperature and heat flux continuities,

$$\mathbf{u}_{f,\mathrm{int}} = \mathbf{u}_{s,\mathrm{int}} \tag{5}$$

$$T_{f,\mathrm{int}} = T_{s,\mathrm{int}} \tag{6}$$

$$\lambda_f \left.\frac{\partial T}{\partial n}\right|_{f,\mathrm{int}} = \lambda_s \left.\frac{\partial T}{\partial n}\right|_{s,\mathrm{int}} \tag{7}$$

where n is the normal direction of the interface, and the subscript "int" represents interface.

Eqs. (1)–(7) can be numerically solved by the finite deference or finite volume methods in classical CFD codes. However, the iteration calculation in CFD may meet unacceptable costs if the geometries are too complex.

2.2. Lattice Boltzmann algorithm

The lattice Boltzmann method provides an alternative way to solve the partial differential equations by evolving variables on a set of lattices [40]. It was proved the lattice BGK method was consistent with the Navier–Stokes equation for fluid flow through Chapman–Enskog expansion [41,42]. Nowadays, the lattice evolution methods have been used to solve most of the

hyperbolic and elliptical equations [43–45]. Here, we present a new discrete evolution equation for heat transfer with a generalized heat source term.

Evolution equation for fluid flow

The two-dimensional 9-speed (D2Q9) lattice Boltzmann model for single time Bhatnagar–Gross–Krook (BGK) relaxation collision operator [40] is used to solve the Navier–Stokes equation for fluid flow

$$f_\alpha(\mathbf{r}+\mathbf{e}_\alpha\delta_t, t+\delta_t) - f_\alpha(\mathbf{r},t) = -\frac{1}{\tau_v}\left[f_\alpha(\mathbf{r},t) - f_\alpha^{\mathrm{eq}}(\mathbf{r},t)\right] \tag{8}$$

with the local equilibrium distribution function

$$f_\alpha^{\mathrm{eq}} = \omega_\alpha \rho_f \left[1 + 3\frac{\mathbf{e}_\alpha\cdot\mathbf{u}}{c^2} + 9\frac{(\mathbf{e}_\alpha\cdot\mathbf{u})^2}{c^4} - \frac{3\mathbf{u}^2}{2c^2}\right] \tag{9}$$

where

$$\omega_\alpha = \begin{cases} 4/9 & \alpha = 0 \\ 1/9 & \alpha = 1,2,3,4 \\ 1/36 & \alpha = 5,6,7,8 \end{cases} \tag{10}$$

$$\mathbf{e}_\alpha = \begin{cases} (0,0) & \alpha = 0 \\ (\cos\theta_\alpha, \sin\theta_\alpha)c, & \theta_\alpha = (\alpha-1)\pi/2 \\ & \alpha = 1,2,3,4 \\ \sqrt{2}(\cos\theta_\alpha, \sin\theta_\alpha)c, & \theta_\alpha = (\alpha-5)\pi/2+\pi/4 \\ & \alpha = 5,6,7,8 \end{cases} \tag{11}$$

and

$$\tau_v = 3\nu\frac{\delta_t}{\delta_x^2} + 0.5 \tag{12}$$

where τ_v is the viscosity-based dimensionless relaxation time, ν is the kinematic viscosity, δ_x is the lattice constant, and δ_t is the time step which is determined by $\delta_t = \delta_x/c$. For gas flows, c takes the value of real sound speed, while for liquid flow, c can take any positive value theoretically only to make τ_v value in $(0.5, 2)$.

After evolving on the discrete lattices, the density and velocity can be calculated using

$$\rho = \sum_\alpha f_\alpha, \qquad \mathbf{u} = \frac{\sum_\alpha \mathbf{e}_\alpha f_\alpha}{\sum_\alpha f_\alpha} \tag{13}$$

Evolution equation for heat transfer

Here we follow Peng's implement [33] which simplifies the thermal lattice Boltzmann model of He et al. [32]. The evolution function for heat transfer without heat source is

$$g_\alpha(\mathbf{r}+\mathbf{e}_\alpha\delta_t) - g_\alpha(\mathbf{r},t) = -\frac{1}{\tau_g}\left[g_\alpha(\mathbf{r},t) - g_\alpha^{\mathrm{eq}}(\mathbf{r},t)\right] \tag{14}$$

Considering the heat source, Jiaung et al. [36,37] gave a scheme for the heat source term quite similar as the external force term treatment in the lattice BGK method. This scheme was proved accurate for heat conduction with phase change. However, it can hardly simulate the convectional heat transfer with heat source. Based on the previous work, here we present a thermal evolution equation with generalized heat source term, which can involve viscous dissipation, pressure compression, and external heat source. Whatever liquid or solid, the evolution equation can be generally given as,

$$g_\alpha(\mathbf{r}+\mathbf{e}_\alpha\delta_t) - g_\alpha(\mathbf{r},t) = -\frac{1}{\tau_g}\left[g_\alpha(\mathbf{r},t) - g_\alpha^{\mathrm{eq}}(\mathbf{r},t)\right] + \omega_\alpha\left(1 - \frac{0.5}{\tau_g}\right)\frac{\dot{Q}}{\rho c_p} \tag{15}$$

where the equilibrium distribution is

$$g_\alpha^{\mathrm{eq}} = \begin{cases} -\varpi_\alpha T\frac{\mathbf{u}^2}{c^2} & \alpha = 0 \\ \varpi_\alpha T\left[\frac{3}{2} + \frac{3}{2}\frac{\mathbf{e}_\alpha\mathbf{u}}{c^2} + \frac{9}{2}\frac{\mathbf{e}_\alpha\mathbf{u}}{c^2} - \frac{3}{2}\frac{\mathbf{u}^2}{c^2}\right] & \\ \qquad\qquad \alpha = 1,2,3,4 & \\ \varpi_\alpha T\left[3 + 6\frac{\mathbf{e}_\alpha\mathbf{u}}{c^2} + 4.5\frac{\mathbf{e}_\alpha\mathbf{u}}{c^2} - 1.5\frac{\mathbf{u}^2}{c^2}\right] & \\ \qquad\qquad \alpha = 5,6,7,8 & \end{cases} \tag{16}$$

with

$$\varpi_\alpha = \begin{cases} -\frac{2}{3} & \alpha = 0 \\ \frac{1}{9} & \alpha = 1,2,3,4 \\ \frac{1}{36} & \alpha = 5,6,7,8 \end{cases} \tag{17}$$

and

$$\tau_g = \frac{3}{2}\frac{\lambda}{\rho c_p c^2 \delta t} + 0.5 \tag{18}$$

where τ_g is the dimensionless relaxation time for energy transport, the value of c in Eqs. (16) and (18) can be independent of that in Eqs. (9) and (11). A larger c may result in a more accurate temperature near the boundary, however, with more computational costs. The temperature can be then calculated by

$$T = \sum_\alpha g_\alpha + \frac{\delta_t}{2}\frac{\dot{Q}}{\rho c_p} \tag{19}$$

For fluid flow, Eqs. (14)–(19) can be used to solve the energy governing equation (3) numerically. For solid, these equations are much simpler due to $\mathbf{u} = 0$.

2.3. Boundary conditions

Boundary treatments for fluid flow

The pressure boundary condition is implemented at the inlet and outlet by introducing an adapted "counter-slip" approach [46,47]. This approach assumes the incoming unknown populations are from an equilibrium distribution with a "counter slip". For inlet as an example, the two unknown parameters, u_x and ρ', are determined by the inlet fluid density,

$$f_1 = \frac{1}{9}\rho'\left(1 + 3\frac{u_x}{c} + 3\frac{u_x^2}{c^2}\right) \tag{20}$$

$$f_5 = \frac{1}{36}\rho'\left(1 + 3\frac{u_x}{c} + 3\frac{u_x^2}{c^2}\right) \tag{21}$$

$$f_8 = \frac{1}{36}\rho'\left(1 + 3\frac{u_x}{c} + 3\frac{u_x^2}{c^2}\right) \tag{22}$$

Substituting Eqs. (20)–(22) into Eq. (13) leads to the solution of the two unknown parameter,

$$u_x = 1 - \frac{f_0 + f_2 + f_4 + 2(f_3 + f_6 + f_7)}{\rho_{\mathrm{in}}} \tag{23}$$

$$\rho' = \rho - \frac{6[f_0 + f_2 + f_4 + f_3 + f_6 + f_7]}{(1 + 3u_x + 3u_x^2)} \tag{24}$$

A similar procedure can be thus applied for the outlet pressure boundary condition. The half way bounce back model is used to treat the fluid–solid interaction in flow, which has second-order accuracy [48].

Boundary treatment for heat transfer

Several boundary treatment methods have been proposed for the thermal lattice Boltzmann evolution equations [49–51]. Here we follow D'Orazio's approach [51] which is consistent with the second-order accurate boundary treatment for fluid flow. In this approach, the incoming unknown populations are also assumed to be equilibrium distribution at a temperature T_0. The value of T_0 is determined by the given constraints, which is the temperature in a Dirichlet boundary, or the heat flux in a Neumann boundary.

For the Dirichlet boundary condition, the unknown distribution functions (North wall, for example) are g_4, g_7, and g_8, which can be obtained by the equilibrium distribution of the local T_0

$$T_0 = 3T_{\text{wall}} - 3T_p + 1.5\delta_t \frac{\dot{Q}_s}{\rho c_p} \tag{25}$$

where T_p is the sum of known populations coming from the internal nodes and nearest wall nodes

$$T_p = g_0 + g_1 + g_3 + g_2 + g_5 + g_6 \tag{26}$$

Thus the unknown distributions are

$$g_\alpha = \varpi_\alpha T_0 \tag{27}$$

For the Newman boundary condition (North wall, for example), the local T_0 is

$$T_0 = 3T_p - 3\chi \frac{\tau_g}{c(\tau_g - 0.5)} \frac{\mathrm{d}T}{\mathrm{d}y} \tag{28}$$

with

$$T_p = g_2 + g_5 + g_6 \tag{29}$$

At the inlet, the unknown distributions are g_1, g_5 and g_8, which can be obtained from the equilibrium distribution of the local T_0

$$T_0 = \frac{6T_{\text{in}} - 6T_p + 3\delta_t \frac{\dot{Q}_f}{\rho c_p}}{2 + 3\frac{u}{c} + 3\frac{u^2}{c^2}} \tag{30}$$

where T_p is the sum of known populations coming from the internal nodes

$$T_p = g_0 + g_2 + g_4 + g_3 + g_6 + g_7 \tag{31}$$

Thus the unknown distributions can be calculated by Eq. (27). For the outlet, the sufficient developed condition is used, which means the distributions at the outlet are set same as the front grid.

Treatment of fluid–solid interface

At the interface, the restriction conditions in Eqs. (5)–(7) have to be added into classical CFD solutions, which increase much computational cost. However in the lattice evolution method, the momentum and energy conservations can be automatically kept by some simplifications. Here we follow the conjugate condition in Ref. [52] and propose a new "half lattice division" scheme as shown in Fig. 1. The interface section is placed not exactly on the lattice nodes but on the middle point between two nodes. Thus, for flow evolution, the half way bounce back model is also applied at the interface for fluid–solid interactions, while for heat transfer evolution, one only need to recognize the local properties of fluid or of solid without any other additional treatments. Although the conjugate condition [52] limits the current method applications to steady cases, however the current scheme actually gains much more computational efficiency with much less coding efforts than classical CFD methods.

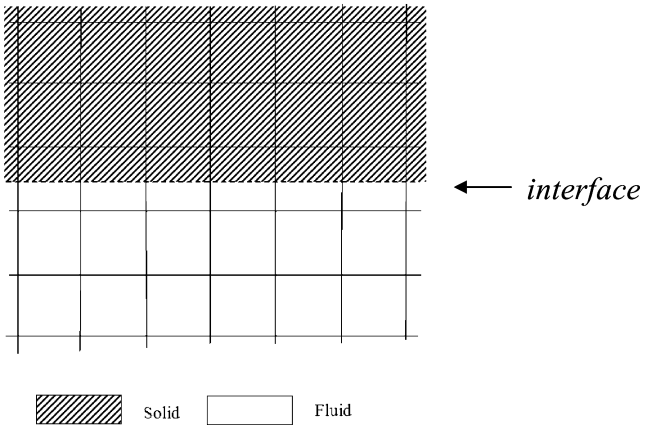


Fig. 1. Half lattice division treatment for the fluid–solid interface.

3. Results and discussion

To validate the present lattice Boltzmann method for conjugate heat transfer, we carried out some simple cases and compared the simulation results with the theoretical solutions. Then a fluid–solid conjugate flow and heat transfer problem was simulated. The results were compared with the classical CFD method.

3.1. Benchmarks

Heat source term

The generalized heat source term was validated at first. Consider a steady Poiseuille flow driven by a pressure gradient $\mathrm{d}P/\mathrm{d}x$. The x direction is periodic. The plate walls are at constant temperature T_w. Therefore, the flow and heat transfer can be simplified as a one-dimensional problem, which has analytical solutions. The viscous dissipation was considered as a heat source $\dot{Q}$ in the energy equation,

$$\dot{Q} = \mu \left(\frac{\mathrm{d}u}{\mathrm{d}y} \right)^2 \tag{32}$$

Thus the temperature distribution is given as

$$T(y) = T_w + \left(\frac{h^2}{2} \frac{\mathrm{d}P}{\mathrm{d}x} \right)^2 \frac{1}{3\lambda\mu} \left[1 - \left(\frac{y}{h} \right)^4 \right] \tag{33}$$

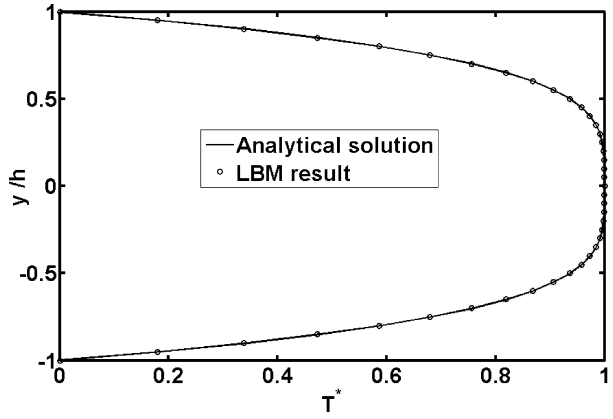


Fig. 2. Comparison of LMB simulated and analytical dimensionless temperatures.

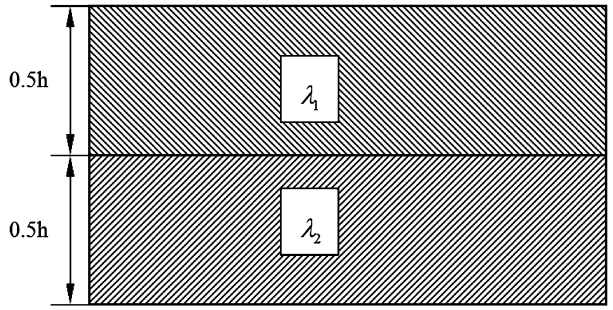


Fig. 3. Sketch of the solid conjugate conduction problem.

where h is half of the channel width.

Fig. 2 shows the comparison of the lattice Boltzmann simulation results and the analytical solution, where the dimensionless temperature is

$$T^* = 3\lambda\mu(T - T_w)\Big/\left(\frac{h^2}{2}\frac{\mathrm{d}P}{\mathrm{d}x}\right)^2 = \left[1 - \left(\frac{y}{h}\right)^4\right] \tag{34}$$

The good agreement between the numerical result and the analytical one indicates that the present generalized heat source treatment can be well used to simulate convection heat transfer with heat source.

Continuities at the interface

For the fluid–solid interface, classical CFD methods use the interface conditions Eqs. (5)–(7) to ensure the continuities of macroscopic parameters and their fluxes. The present lattice Boltzmann method just uses a "half lattice division" (see Fig. 1) to deal with the interface.

Consider a pure heat conduction problem between two solids shown in Fig. 3. The thermal conductivities are different, and $\lambda_1 : \lambda_2 = 1 : 10$. Both the top and bottom of the domain are Dirichlet boundary conditions at T_1 and T_2 (assuming $T_1 > T_2$). If the other direction is adiabatic, the problem has analytical solution,

$$T(y) = \begin{cases} T_2 + \frac{2y(T_1 - T_2)}{11h} & 0 \leqslant y \leqslant 0.5h \\ \frac{20T_2 - 9T_1}{11} + \frac{20y(T_1 - T_2)}{11h} & 0.5h \leqslant y \leqslant h \end{cases} \tag{35}$$

Fig. 4 compares the lattice Boltzmann simulation results with the analytical solution. They agree quite well, which indicates that the lattice Boltzmann method with the present "half lattice division" treatment for the interface is effective. The continuous temperature and heat flux are obtained by the lattice Boltzmann method without any additional restrict at the interface. This characteristic will bring great convenience for modeling complex geometry problems.

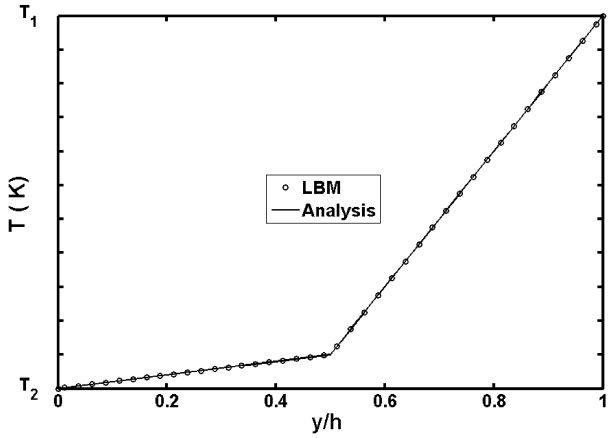


Fig. 4. Comparison of LBM simulated and analytical temperature profiles.

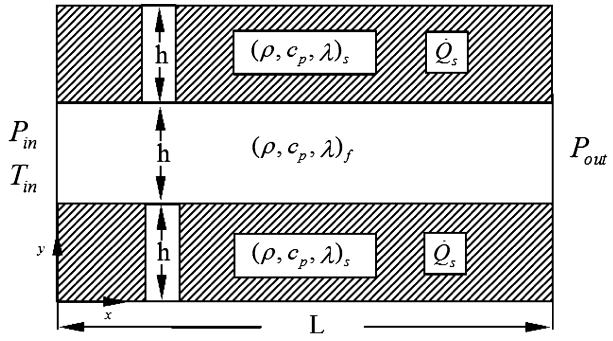
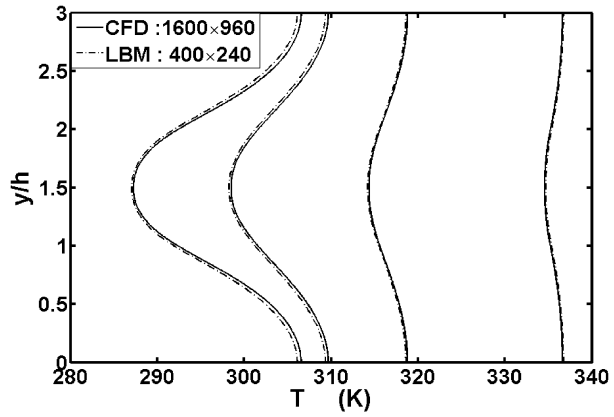


Fig. 5. Geometry and boundary conditions of the flow and heat transfer in a microchannel.

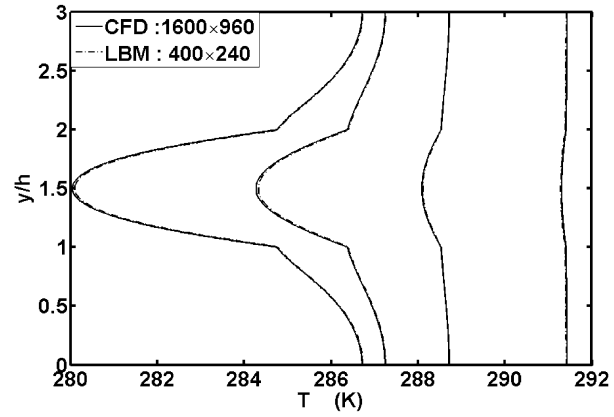
3.2. Application for microchannel convection

In most cases of macroscale channel flows, the channel wall thickness is relative thin to the channel width. The boundary condition for the energy equation can be simplified as constant temperature or constant heat flux on the wall. However in microscale flows, the wall thickness becomes comparable to the channel width and cannot be treated as thin wall [8]. The axial heat conduction in the wall is of strong effect on the heat transfer coefficients [53]. The fluid–solid conjugate heat transfer must be taken into consideration. In fact, the flow and heat transfer in complex micro channels have very important applications in bio- and medical engineering [54,55].

Here let us consider the two-dimensional flow and heat transfer in a simple heated thick–wall channel as shown in Fig. 5. The wall thickness equals to the channel width h. The channel is L in length, which is 5 times of h in current computational cases. The fluid is driven by a pressure difference between inlet and outlet. The outer surfaces of the wall are adiabatic. The uniform heat generation in the wall is $\dot{Q}_s$. The



(a) $\lambda_s : \lambda_f$ =1:1



(b) $\lambda_s : \lambda_f$ =10:1

Fig. 6. Temperature profiles at positions of $x/L = 1/16$, $1/8$, $1/4$, and $1/2$ simulated by LBM and CFD.

Table 1
Simulation parameters

ρ_f [kg m^{-3}]	1.225	ρ_s [kg m^{-3}]	2719
$c_{p,f}$ [J kg^{-1} K^{-1}]	1006.43	$c_{p,s}$ [J kg^{-1} K^{-1}]	871
λ_f [W m^{-1} K^{-1}]	0.0242	$\lambda_{s,1}$ [W m^{-1} K^{-1}]	0.0242
μ_f [kg m^{-1} s^{-1}]	1.789×10^{-5}	$\lambda_{s,2}$ [W m^{-1} K^{-1}]	0.242
$\dot{Q}_s$ [W m^{-3}]	1×10^9	ΔP [Pa]	5.0
T_{in} [K]	273.15	h [m]	0.6×10^{-5}

coming fluid is isothermal at T_{in}. The outlet is assuming to be fully developed in heat transfer. The other parameters are listed in Table 1.

Two cases were investigated by taking the ratio values of solid thermal conductivity to fluid thermal conductivity $\lambda_s : \lambda_f$ equal 1 : 1 and 10 : 1. If the solid thermal conductivity is much larger than 10 times of the fluid thermal conductivity, the temperature drop in solid can be ignored comparing with that in fluid. Fig. 6 shows the temperature profiles at four positions. The lattice Boltzmann method predictions are compared with the classical CFD simulation results in the same figures. The classical CFD solved the governing equations (1)–(7) by the finite volume method (FVM), which was implemented on the code Fluent 6.02 [56].

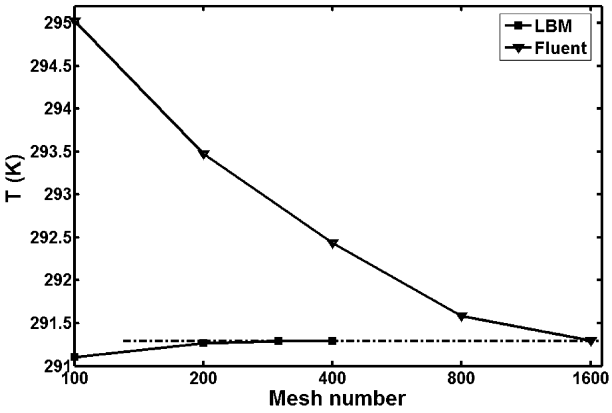


Fig. 7. Temperature variations at the middle point ($x/L = 0.5$ and $y/h = 1.5$) by CFD and LBM with the mesh number in x direction.

Both the lattice Boltzmann method and the classical CFD method need to check the mesh size independency. We found when the mesh number is larger than 1500 for CFD or 200 for LBM in x direction, the relative errors were below 0.1%. Fig. 6 compares the results between CFD at 1600×960 and LBM at 400×240 mesh divisions. The results agree very well, which indicates the present lattice Boltzmann algorithm can be used to deal with the fluid–solid conjugate heat transfer problems.

Comparing with the classical CFD method, the present LBM is more efficient in calculation. Fig. 7 compares the calculated temperature variations at the middle point position vs. the mesh division number in x direction. The results show the classical CFD method needs much finer mesh than the lattice Boltzmann method to get a same accuracy. For such a simple geometry case, the computational time for CFD is over twice of that for LBM. The advantage of the present method will be much more significant for complex geometry systems and for parallel computing.

4. Conclusions

A lattice Boltzmann algorithm for fluid–solid conjugate heat transfer is developed. A new generalized heat generation implement is presented and validated. For the fluid–solid interface, a "half lattice division" treatment is proposed to deal with fluid–solid interaction and energy transport, which insures temperature and heat flux continuities at the interface. The new scheme is applied to simulate flow and heat transfer in a heated thick–wall microchannel. The results agree very well with the predictions of classical CFD, however, with much less computational costs. The current algorithm is quite suitable for numerical analysis of fluid–solid conjugate heat transfer problem in complex geometry.

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